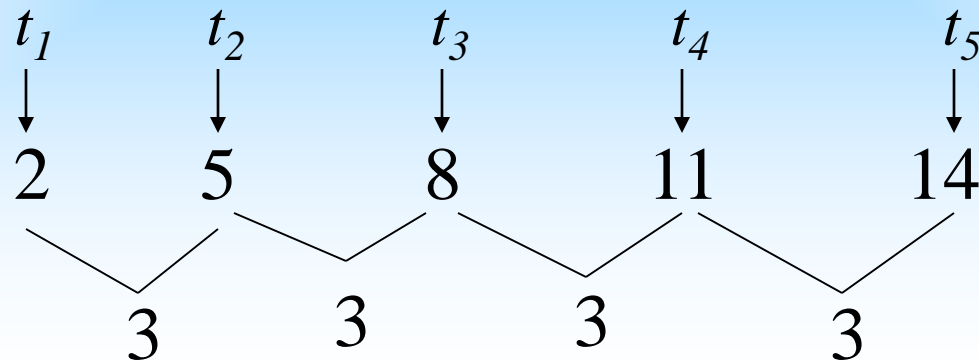# Arithmetic Sequences

Definition

A sequence in which a constant (d) can be added to each term to get the next term is called an **Arithmetic Sequence**.

The constant (d) is called the Common Difference.

To find the common difference (d), subtract any term from one that follows it.

t_1 t_2 t_3 t_4 t_5

2   5   8   11   14

3   3   3   3

Examples:

Find the first term and the common difference of each arithmetic sequence.

1.) $4, 9, 14, 19, 24$

   First term (a): 4
   Common difference (d): $a_2 - a_1 = 9 - 4 = 5$

2.) $34, 27, 20, 13, 6, -1, -8, ...$

   First term (a): 34
   Common difference (d): -7

BE CAREFUL: _ALWAYS_ CHECK TO MAKE SURE THE DIFFERENCE IS THE SAME BETWEEN EACH TERM !

Now you try!

Find the first term and the common difference of each of these arithmetic sequences.

a) 1, -4, -9, -14, ….

b) 11, 23, 35, 47, ….

Answers with solutions

a) 1, -4, -9, -14, ….   a = 1   *and*

$d = a_2 - a_1 = -4 - 1 = -5$

b) 11, 23, 35, 47, ….   a = 11   *and*

$d = a_2 - a_1 = 23 - 11 = 12$

The first term of an arithmetic sequence is (a). We add (d) to get the next term. There is a pattern, therefore there is a formula we can use to give use any term that we need without listing the whole sequence ☺.

3, 7, 11, 15, ….   We know a $= 3$ and d $= 4$

$t_1 = a = 3$

$t_2 = a+d = 3+4 = 7$

$t_3 = a+d+d = a+2d = 3+2(4) = 11$

$t_4 = a+d+d+d = a+3d = 3+3(4) = 15$

The first term of an arithmetic sequence is (a). We add (d) to get the next term. There is a pattern, therefore there is a formula (explicit formula) we can use to give use any term that we need without listing the whole sequence ☺.

The nth term of an arithmetic sequence is given by:

$$t_n = a + (n - 1) d$$

The last # in the sequence/or the # you are looking for

First term

The position the term is in

The common difference

Examples: Find the 14th term of the arithmetic sequence 4, 7, 10, 13,......

$$t_n = a + (n-1)\,d$$

$$t_{14} = 4 + (14-1)3$$

$$= 4 + (13)3$$

$$= 4 + 39$$

$$= 43$$

You are looking for *the term!*

The 14th term in this sequence is the number 43!

Now you try!

Find the 10th and 25th term given the following information. Make sure to derive the general formula first and then list ehat you have been provided.

a) 1, 7, 13, 19 ….

b) The first term is 3 and the common difference is -21

c) The second term is 8 and the common difference is 3

Answers with solutions

a) 1, 7, 13, 19 …. ….

$a = 1$ and $d = a_2 - a_1 = 7 - 1 = 6$

$t_n = a + (n-1)d = 1 + (n-1)\,6 = 1 + 6n - 6$ So $t_n = $ **6n-5**

$t_{10} = 6(10) - 5 = 55$

$t_{25} = 6(25) - 5 = 145$

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b) The first term is 3 and the common difference is -21

$a = $ **3** and $d = $ **-21**

$t_n = a + (n-1)d = 3 + (n-1)\,-21 = 3 - 21n + 21$ So $t_n = $ **24-21n**

$t_{10} = 24 - 21(10) = -186$ $t_{25} = 24 - 21(25) = -501$

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c) The second term is 8 and the common difference is 3

$a = 8 - 3 = $ **5** and $d = $ **3**

$t_n = a + (n-1)d = 5 + (n-1)\,3 = 5 + 3n - 3$ So $t_n = $ **3n+2**

$t_{10} = 3(10) + 2 = 32$ $t_{25} = 3(25) + 2 = 77$

Examples:

Find the 14th term of the arithmetic sequence with first term of 5 and the common difference is –6.

$a = 5$ and $d = -6$

$$t_n = a + (n - 1)\, d$$

$$t_{14} = 5 + (14 - 1)\text{-}6$$

$$= 5 + (13) * \text{-}6$$

$$= 5 + \text{-}78$$

$$= \text{-}73$$

You are looking for *the term! List which variables from the general term are provided!*

The 14th term in this sequence is the number -73!

Examples:

In the arithmetic sequence 4,7,10,13,..., which term has a value of 301?

$$t_n = a + (n - 1)\, d$$

$$301 = 4 + (n - 1)3$$

$$301 = 4 + 3n - 3$$

$$301 = 1 + 3n$$

$$300 = 3n$$

$$100 = n$$

You are looking for *n!*

The 100th term in this sequence is 301!

Examples:

In an arithmetic sequence, term 10 is 33 and term 22 is –3. What are the first four terms of the sequence?

$t_{10}=33$

$t_{22}= -3$

Use what you know!

For term 10: $t_n = a + (n – 1) d$
$$33= a + 9d$$

For term 22: $t_n = a + (n – 1) d$
$$-3= a + 21d$$

HMMM! Two equations you can solve!

SOLVE:
$$33 = a+9d$$
$$-3 = a+21d$$

By elimination
$$-36 = 12d$$
$$-3 = d$$

SOLVE:
$$33 = a + 9d$$
$$33 = a +9(-3)$$
$$33 = a –27$$
$$60 = a$$

The sequence is 60, 57, 54, 51, …….

Arithmetic Sequences

Every day a radio station asks a question for a prize of $150. If the 5th caller does not answer correctly, the prize money increased by $150 each day until someone correctly answers their question.

Arithmetic Sequences

Make a list of the prize amounts for a week (Mon - Fri) if the contest starts on Monday and no one answers correctly all week.

Arithmetic Sequences

- Monday : $150
- Tuesday: $300
- Wednesday: $450
- Thursday: $600
- Friday: $750

Arithmetic Sequences

- These prize amounts form a **sequence**, more specifically each amount is a term in an **arithmetic sequence**. To find the next term we just **add** $150.