Warm-Up   1/22/18

Bring out your Linear Models Packet and complete the Bath Tub Problem.

Agenda for Today   1/22/18

1. Complete Bath Tub Problem
2. Shoe Size Problem HW
3. Characteristics of Linear Functions
4. Arithmetic Sequences (if time permits)
5. 1st Block: Text @82d4g to 81010
6. 2nd Block: Text @h3c272 to 81010
7. 3rd Block: Text @a88a9d to 81010

1. Bathtub Problem: You pull out the plug from the bathtub. After 40 seconds, there are 13 gallons of water left in the tub. One minute after you pull the plug, there are 10 gallons left. Assume that the number of gallons varies linearly with the time since the plug was pulled.

$(40, 13)$, $(60, 10)$

a. Write the particular equation expressing the number of gallons (g) left in the tub in terms of the number of seconds (s) since you pulled the plug.   Hint: To find the equation you must first find what? Slope

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 13}{60 - 40} = \boxed{\frac{-3}{20}}$$

$$y - y_1 = m(x - x_1)$$ $(60, 10)$

$$y - 10 = \frac{-3}{20}(x - 60)$$

$$y - 10 = \frac{-3}{20}x + 9$$
$$\qquad + 10$$

$$\boxed{y = \frac{-3}{20}x + 19} \implies$$

3

b. How many gallons would be left after 20 seconds? 50 seconds?

$$f(20) = \frac{-3}{20}(20) + 19$$

$$f(20) = -3 + 19$$

$$f(20) = 16$$

$$(20, 16)$$

$$f(50) = 11.5 \text{ gallons} \quad (50, 11.5)$$

c. At what time will there be 7 gallons left in the tub?

$$\underset{-19}{\overset{x}{7}} = \frac{-3}{20}x + 19 \qquad 20(-12) = \frac{-3}{20}x \cdot 20$$

$$\frac{-240}{-3} = \frac{-3x}{-3} \qquad x = 80 \, sec$$

d. Find the y-intercept (gallon-intercept). What does this number represent in the real world?

$$y = \frac{-3}{20}(0) + 19 = 19 \, gallons$$

The initial amount of water in the bath tub.

e. Find the x-intercept (time- intercept). What does this number represent in the real world?

$$0 = \frac{-3}{20}x + 19$$

$$\frac{-19}{-19} \qquad -19$$

$$\overset{20}{-19} = \frac{-3}{20}x \cdot 20$$
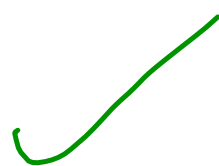
$$\frac{-380}{-3} = \frac{-3x}{-3}$$

$$x = 126.6 \approx 127 \, seconds$$

It takes approximately 127 seconds for the bath tub to be empty.

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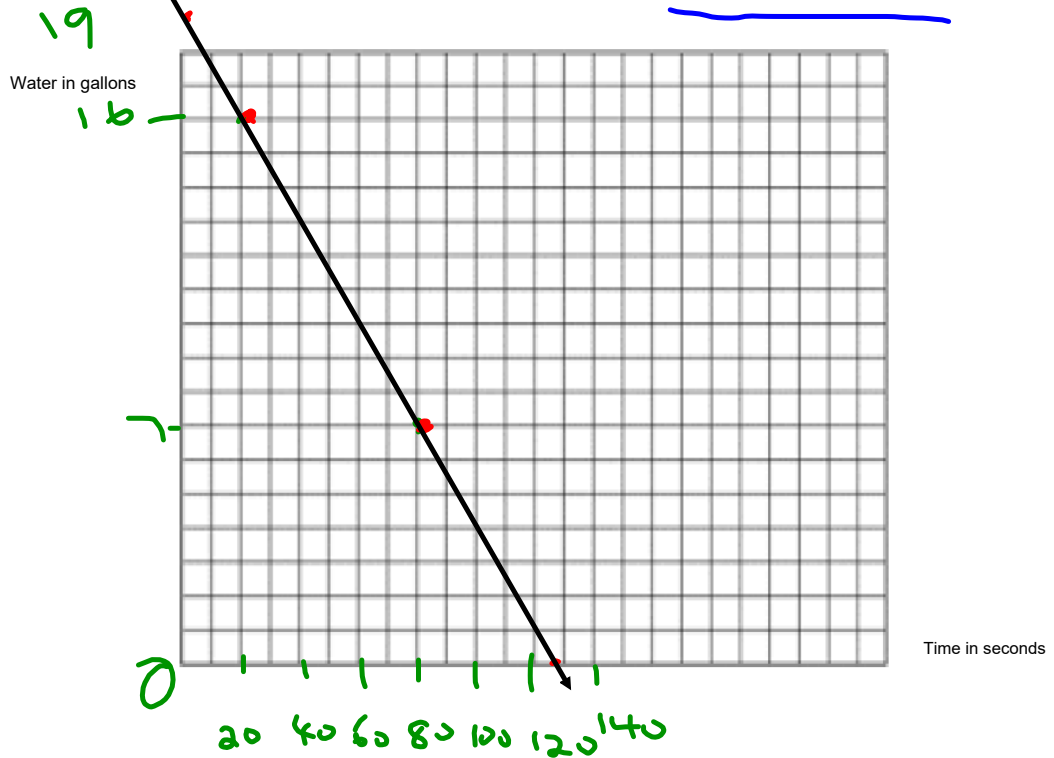$(40, 13)$

$(60, 10)$

$(20, 16)$ ✓

$(50, 11.5)$

$(80, 7)$ ✓

$(0, 19)$ ✓

$(127, 0)$ ✓

f. Plot the graph of this linear function. Use a suitable domain.

19

Water in gallons

16

7

0

20 40 60 80 100 120 140

Time in seconds

g. What is the slope? $\dfrac{-3}{20}$ What does this number 7 represent?

For every 20 seconds, the amount of water reduces by 3 gallons.

2. Shoe Size Problem: The size of a shoe a person needs varies linearly *y* with the length of his or her foot. The smallest adult shoe size is Size 5, *x* and fits a 9-inch long foot. An 11-inch long foot takes a Size 11 shoe.

(9, 5) , (11, 11)

FUNCTIONS TASK – FOUNDATIONS OF ALGEBRA

a. Write the particular equation expressing shoe size (s) in terms of foot length (l).

b. If your foot is a foot long, what size do you need? _____

c. Bob Lanier, who once played in the NBA wears a Size 22. How long is his foot? _____
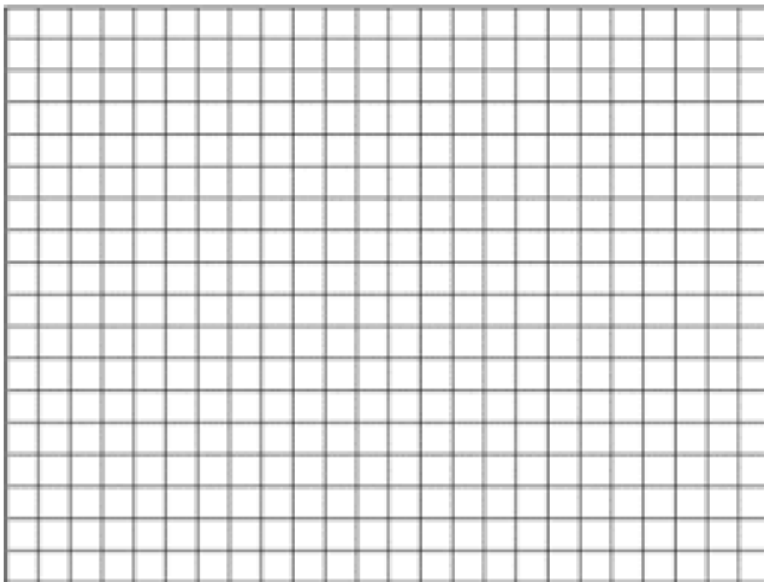
d. Find the shoe-size-rate intercept. (Y-intercept)_____
What does this number represent in the real world?

_____

_____

e. Calculate the foot-length-intercept. (X-intercept) _____
What it tell you about the real world?

_____

_____

f. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

g. What is the slope? What does this number 11 represent?

Characteristics of Linear Functions 1/22/18

Characteristics of Linear Functions (pp. 1 of 8) KEY

Parent Function

Linear Parent Function:

$y = x$

Domain:
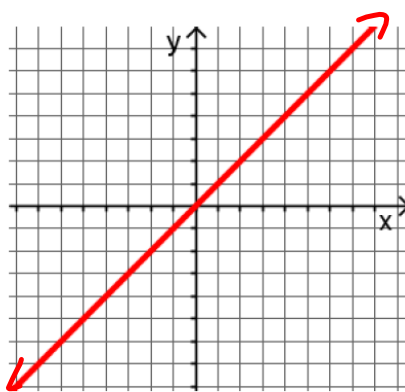$(-\infty, \infty)$ $\mathbb{R}$
All real numbers

Range:
$(-\infty, \infty)$ $\mathbb{R}$
All real numbers

Table

x	y
-3.5	-3.5
-1	-1
0	0
1	1
4.5	4.5
6	6

- What patterns do you observe in the table and graph of the linear parent function?
For each point, the *x*-values and *y*-values are equal. As *x* increases, *y* increases. Fractional values are possible.

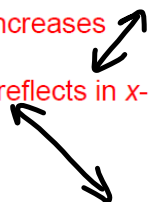| **Review: Characteristics** | Characteristics of linear functions (such as slope, intercepts, and equations) have been addressed in previous math courses. |

Forms of Linear Equations

Standard Form:	$ax + by = c$ (No fractions and a is positive value)
Point-Slope Form:	$y - y_1 = m(x - x_1)$
Slope-Intercept Form:	$y = mx + b$

Slope-Intercept Form

In the slope-intercept form, explain how changes in m (the slope) and b (the y-intercept) affect the graph of the parent function.

Changes caused by <u>m</u>	Changes caused by <u>b</u>
• $m = 1$, no change in slope of parent function • $m > 0$, m is positive and the line increases from left to right • $m < 0$, m is negative and the line reflects in x-axis; decreases from left to right	• $b = 0$, no change in y-intercept of parent function, stays at (0, 0) • $b > 0$, vertical shift up "b" units • $b < 0$, vertical shift down "b" units

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Characteristics of Linear Functions (pp. 2 of 8) KEY

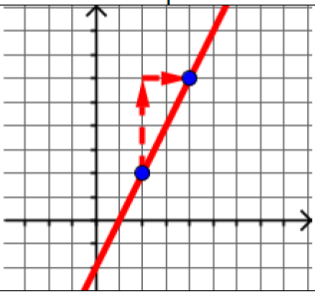**Finding Slope**

How would you define _slope_?
A line's rate of change
A measure of a line's "steepness" or "slant"

Explain how to find _slope_ from each representation.

Graph	Table	Two Points	Equation
	$\begin{array}{cc} x & y \\ 2 & 5 \\ 4 & 19 \\ 7 & 40 \end{array}$ (here, $m = 7$)	$(-1,3)$ and $(4, -5)$ (here, $m = -\frac{8}{5}$)	A) $y = -2x + 4$ (here, $m = -2$) B) $y = -5 + \frac{2}{3}x$ (here, $m = \frac{2}{3}$) C) $3x + y = 8$ (*here, $m = -3$)
Count: $\dfrac{\text{Rise}}{\text{Run}}$ (here, $m = \frac{4}{2} = 2$)	Find $\dfrac{\Delta y}{\Delta x}$ or $\dfrac{\text{change in } y}{\text{change in } x}$	Compute $\dfrac{y_2 - y_1}{x_2 - x_1}$	Look For the coefficient of the 'x' term (as long as the equation is in $y = mx+b$ form*)

Finding Intercepts

What is an *intercept*?

<u>Point where a line intersects (or crosses) the x-axis or y-axis</u>

Explain how to find a **y-intercept** from each representation.

From the graph:	Determine where the line would cross the <u>y-axis</u>
From a table:	Look for the point <u>(0, y)</u>
From the $y = mx + b$ equation:	Identify the <u>b-value</u>
From any equation:	Plug in <u>x = 0</u>, then solve for <u>y</u>

Explain how to find an **x-intercept** from each representation.

From the graph:	Determine where the line would cross the <u>x-axis</u>
From a table:	Look for the point <u>(x, 0)</u>
From an equation:	Plug in <u>y = 0</u>, then solve for <u>x</u>

Characteristics of Linear Functions (pp. 3 of 8) KEY

Finding Equations

There are several ways to determine the equation of a line, depending on the given information.

If you have…	Then plug values into…
The slope and y-intercept	Slope-intercept Form : $y = mx + b$
The slope, and coordinates for a point	Point-slope Form : $y - y_1 = m(x - x_1)$
Coordinates for two points	$\dfrac{y_2 - y_1}{x_2 - x_1}$ *then* $y - y_1 = m(x - x_1)$
A table of data	A calculator's lists (to do a linear regression)

Special Lines and Slopes

| Lines	Slopes	Sample Equation(s)
Horizontal Lines	A horizontal line has a slope of zero	$y = \#$
Vertical Lines	A vertical line has undefined slope	$x = \#$
Parallel Lines	Slope of parallel lines are equal ($m_1 = m_2$)	Like $y = 2x + 3$ and $y = 2x - 7$
Perpendicular Lines	Slopes of perpendicular lines are opposite reciprocals $\left(-\left(\frac{1}{m_1}\right) = m_2\right)$	Like $y = 2x + 3$ and $y = -\frac{1}{2}x - 7$

The reciprocal of $\frac{1}{3} = 3$

The reciprocal of $4 = \frac{1}{4}$

Sample Problems   Provide information about each linear function.

1.

x	y
−1	−7
1	−3
2	−1
3	1
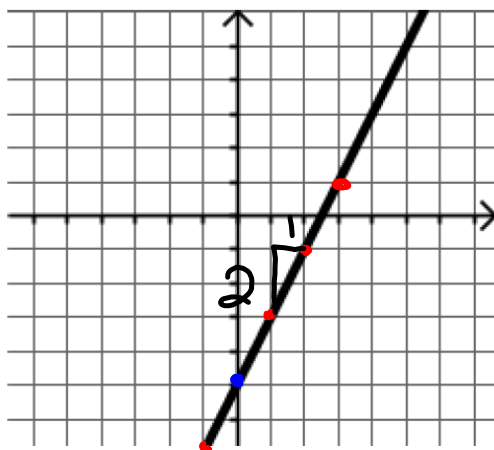
A) Fill in the table of values.
B) Determine the slope.

$$m = 2$$

C) Find the equation for the line.
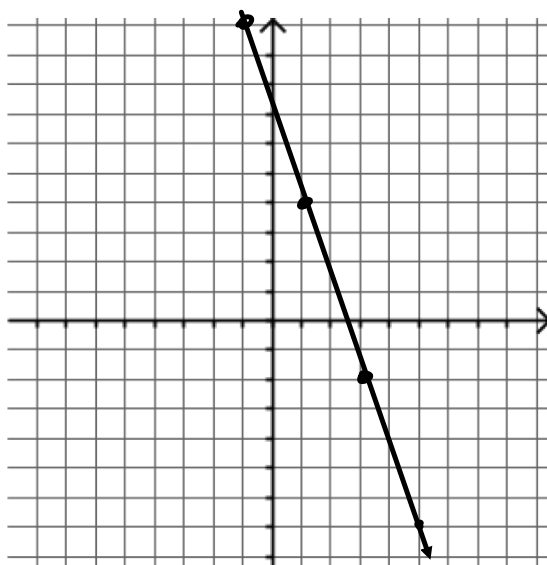
$$y = 2x - 5$$

D) What are the intercepts?

y-int: $(0, -5)$

x-int: $(2.5, 0)$

Characteristics of Linear Functions (pp. 4 of 8)

2.

x	y
-1	10
1	4
3	-2
5	-8

A) Graph the function.

B) Determine the slope.

$$m = \frac{-6}{2} = -3$$
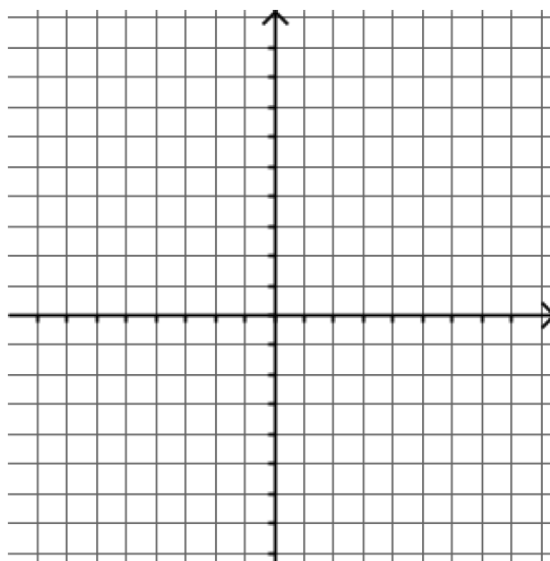
C) Find the equation for the line.

$$y = -3x + 7$$

D) What are the intercepts?

y int: $(0, 7)$

x int: $(2.5, 0)$

HW

3.

x	y

Equation: $y = -\frac{2}{3}x + 4$

A) Complete a table of values.
B) Graph the function.
C) Write down the slope.

D) What are the intercepts?

#4-10
Hw

Characteristics of Linear Functions (pp. 5 of 8)

From the given information, find the equation of the line both in slope-intercept and standard forms.

4. The slope of $-\dfrac{1}{2}$ and contains point (–2, 5)

5. Contains points (2, -3) and (-6, 1)

6. Contains point (0, 4) and is parallel to $y = 2x - 3$

7. Contains point (-4, 5) and is perpendicular to $2x + 3y = 7$

8. Contains (5, 1) and is perpendicular to $y = 3$.
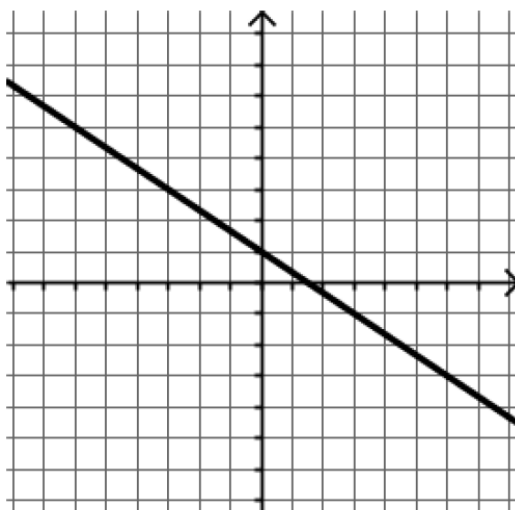
9. Contains (5, 1) and is parallel to $y = 3$.

10. Contains (-2, -7) and is perpendicular to $x = 4$.

Characteristics of Linear Functions (pp. 6 of 8)

Practice Problems

1.

x	y

A) Fill in the table of values.
B) Determine the slope.

C) Find the equation for the line.

D) What are the intercepts?

2.

x	y
-6	3
-3	4
3	6
6	7
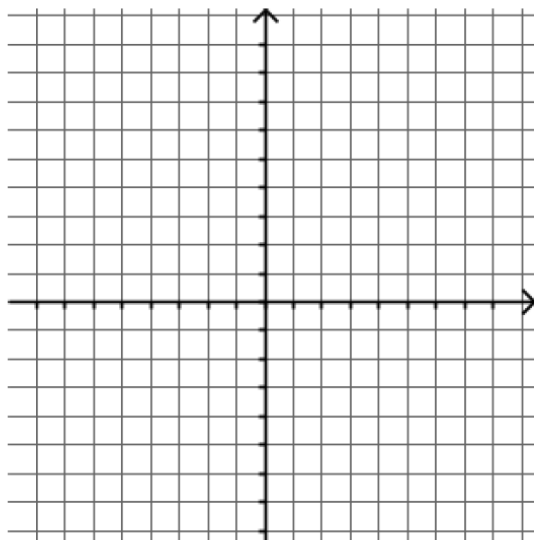
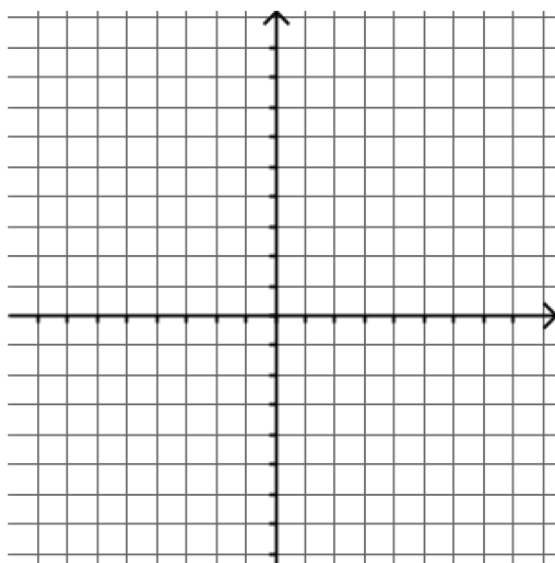A) Graph the function.
B) Determine the slope.

C) Find the equation for the line.

D) What are the intercepts?

Characteristics of Linear Functions (pp. 7 of 8)

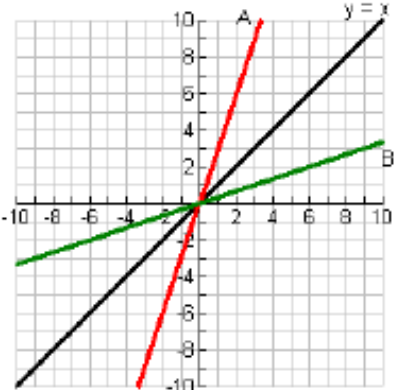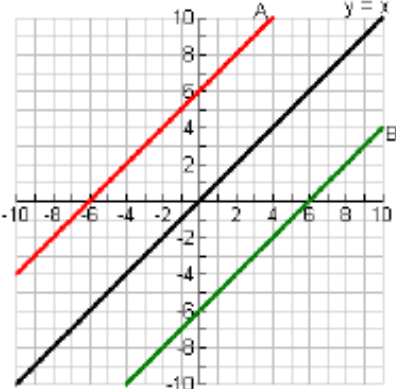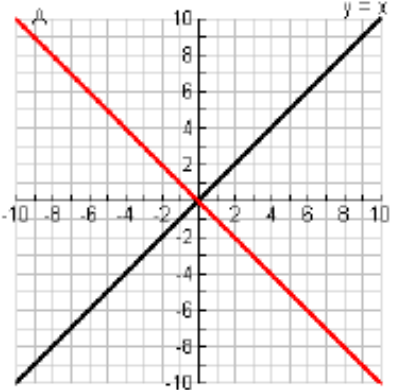3.

x	y

Equation: $y = \frac{3}{2}x - 5$

A) Complete a table of values.
B) Graph the function.
C) Write down the slope.

D) What are the intercepts?

Given information, find the equation of the line in *y*-intercept and standard forms.

4. The slope of –3 and contains the point (0, 3)

5. The slope of $\dfrac{5}{3}$ and contains point (-6, -2)

6. Contains points (4, -1) and (-2, -13)

7. Contains point (-1, 2) and is parallel to $x – 2y = -3$

8. Contains point (5, -3) and is perpendicular to $y = 5x - 4$

9. Contains (-4, 3) and is perpendicular to $y = 2$.

10. Contains (8, -1) and is parallel to $y = 1$.

11. Contains (-2, -3) and is perpendicular to $x = 2$.

12. For each of the linear functions on the graphs below, compare it to the linear parent function in terms of vertical shifts and vertical compressions. Identify the parameter that determines the change and determine the function rule.

Graph	Transformations/Changes	Equations
	A) Vertical stretch by a factor of 3, m = 3	A) y = 3x
	B) Vertical compression by a factor of $\frac{1}{3}$, $m = \frac{1}{3}$	B) $y = \frac{1}{3}x$
	A) Vertical shift up by 6 units, b = 6	A) y = x + 6
	B) Vertical shift down by 6 units, b = -6	B) y = x - 6
	Reflection over the x-axis, m = -1	y = -x

Student-Led Closing:

What did you learn today?