DAY 10: APPLICATIONS OF THE VERTEX

Unit 3B: Quadratic Functions

3/8/18

How Do I know a Given Word Problem is Talking About the Vertex?

Words that Indicate Finding Vertex	**Quadratic Equations**	
• Minimum/Maximum • Minimize/Maximize • Least/Greatest • Smallest/Largest	Standard Form: $y = ax^2 + bx + c$ Vertex Form: $y = a(x-h)^2 + k$ Factored Form: $y = a(x-p)(x-q)$ Vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	y-int: $(0, c)$ vertex: (h, k) x-int: $(p, 0)$ & $(q, 0)$

Example 1

The arch of a bridge forms a parabola modeled by the function:

$y = -0.2(x - 40)^2 + 25$, *where x is the horizontal distance (in feet) from the arch's left end and y is the corresponding vertical distance (in feet) from the base of the arch. How tall is the arch?*

Vertex = (40, 25)

The arch is 25ft high.

Example 2

Suppose the flight of a launched bottle rocket can be modeled by the equation $y = -x^2 + 6x$, where y measures the rocket's height above the ground in meters and x represents the rocket's horizontal distance in meters from the launching spot at x = 0.

$a = -1 \quad b = 6 \quad c = 0$

a. How far has the bottle rocket traveled horizontally when it reaches it maximum height? What is the maximum height the bottle rocket reaches?

b. How far does the bottle rocket travel in the horizontal direction from launch to landing?

The height when it lands is zero.

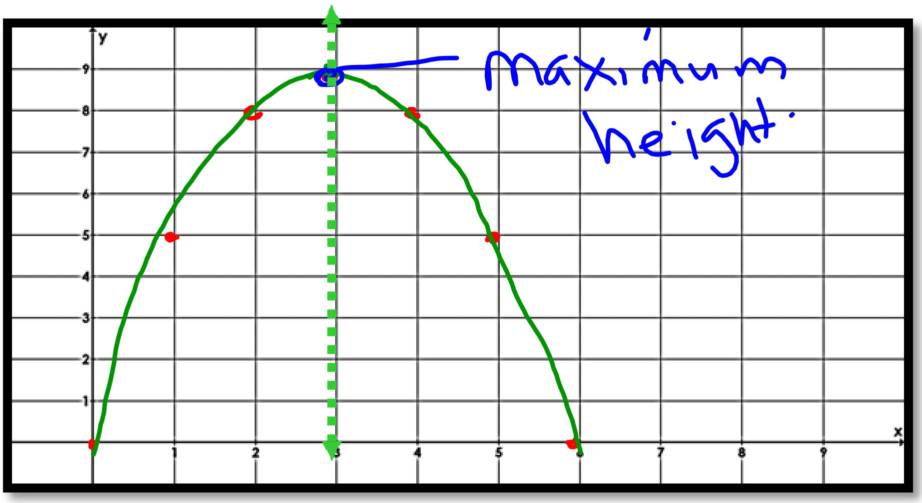
So, $-x^2 + 6x = 0$

$$-x(x-6) = 0$$

$$x = 0, \quad x - 6 = 0$$

$$x = 6$$

The bottle rocket travels in the horizontal direction for 6 meters from launch to landing.



$$x = \frac{-b}{2a} = \frac{-6}{2(-1)} = \frac{-6}{-2} = 3$$

$$f(3) = -(3)^2 + 6(3)$$

$$= -9 + 18$$

$$f(3) = 9$$

The bottle rocket traveled horizontally for 3 meters reaching a maximum height of 9 meters.

Example 3

A frog is about to hop from the bank of a creek. The path of the jump can be modeled by the equation:
$h(x) = -x^2 + 4x + 1$, where h(x) is the frog's height above the water and x is the number of seconds since the frog jumped. A fly is cruising at a height of 5 feet above the water. Is it possible for the frog to catch the fly, given the equation of the frog's jump?

$a = -1, b = 4, c = 1$

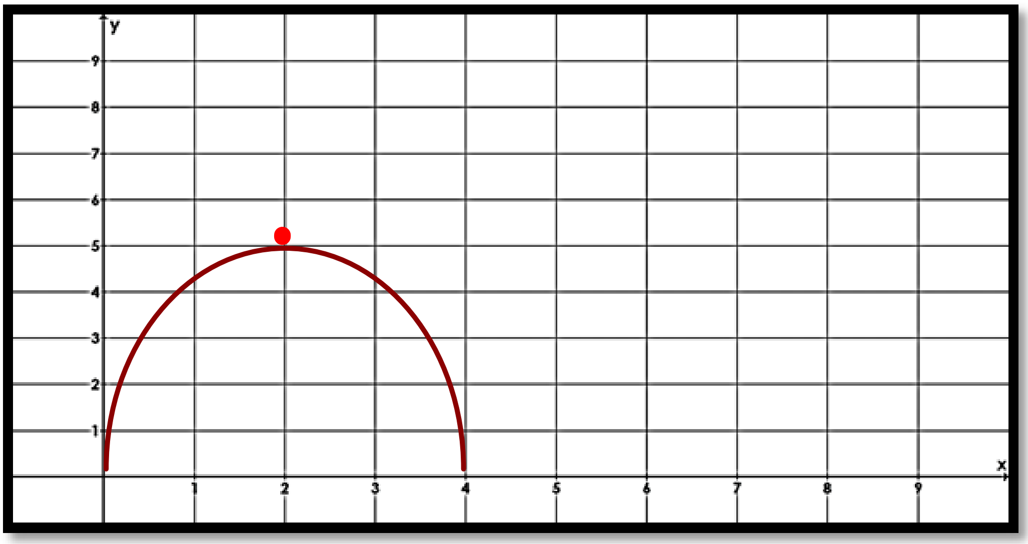
$x = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

$f(2) = -(2)^2 + 4(2) + 1$

$f(2) = -4 + 8 + 1$

$f(2) = 5 \text{ ft}$

Vertex $= (2, 5)$



Yes, it is possible for the frog to catch the fly.

Example 4

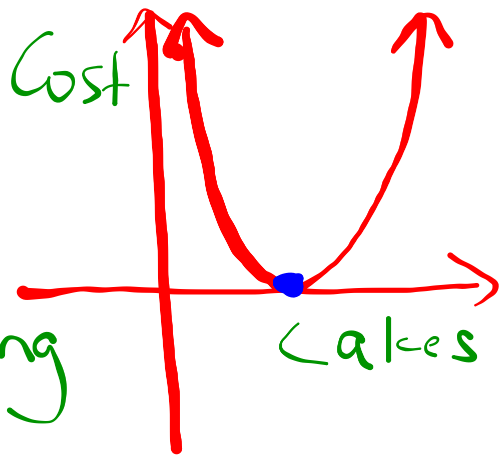
A baker has modeled the monthly operating costs for making wedding cakes by the function
$y = 0.5x^2 - 12x + 150$, where y is the total costs in dollars and x is the number of cakes prepared.

$a = 0.5 \quad b = -12$

a. How many cakes should be prepared each month to yield the minimum operating cost?

$$x = \frac{-(-12)}{2(0.5)} = \frac{12}{1} = 12$$

12 cakes will yield the minimum operating cost.



b. What is the minimum monthly operating cost?

$f(12) = 0.5(12)^2 - 12(12) + 150$

$f(12) = 78$

The minimum operating cost is \$78.

5. A street vendor sells about 20 shirts a day when she charges $8 per shirt. If she decreases the price by $1, she sells about 10 more shirts each day.

a. How many shirts does she have to sell to maximize her revenue? What is her maximum revenue?

$ Price	Number of Shirts Sold	Revenue
9	10	90
$8	20	160
7	30	210
6	40	240
5	50	250
4	60	240
3	70	210
2	80	160

70, 50, 30, 10, -10, -30, -50

-20, -20, -20, -20, -20, -20

b. How much more will she make a day?

It is different each day.
For instance, she makes $70 more in the 2nd day, $50 in the 3rd day, etc.

c. Write a quadratic function that models the scenario.

To write the function in Standard form, $y = ax^2 + bx + c$,
$a = 1/2$ of the 2nd difference
So $a = 1/2(-20) = \boxed{-10}$

To Find the "b", Choose any 2 points (x-price) and (y-revenue)
eg- (9, 90)

$90 = -10(9)^2 + 9b$

$90 = -81\cancel{0} + 9b$

$+810 \quad +8\cancel{1}0$

$\frac{900}{9} = \frac{9b}{9}$

$\boxed{b = 100}$

Our Function is

$$\boxed{y = -10x^2 + 100x}$$

6. You run a canoe rental business on a small river in Georgia. You currently charge $12 per hour canoe and average 36 rentals a day. An industry journal says that for every fifty cent increase in rental price, the average business can expect to lose two rentals a day.

a. Use this information to attempt to maximize your income. What should you charge?

\$ Price X	Number of Rentals	Revenue Y
11	40	440
11.50	38	437
\$12	36	432
12.50	34	425
13.00	32	416
13.50	30	405
14.00	28	392
14.50	26	377

b. Write a quadratic function that models the scenario.

2nd difference = −2

$a = \frac{1}{2}(-2) = \boxed{-1}$

Use (11, 440)

$440 = -1(11)^2 + 11(b)$

$440 = -121 + 11b$

$+121 \quad +121$

$\frac{561}{11} = \frac{11b}{11}$

$b = 51$

Function: $y = -x^2 + 51x$

Quadratic Key Words