Unit 3A – Solving Quadratic Equations by Factoring

A quadratic equation is written in the **Standard Form**,

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers and $a \neq 0$.

Examples:

$x^2 - 7x + 12 = 0$ (standard form)

$x(x+7) = 0$

$3x^2 + 4x = 15$

Unit 3A – Solving Quadratic Equations by Factoring

Zero Factor Property:

If a and b are real numbers and if $ab = 0$,

then $a = 0$ or $b = 0$.

Examples:

$$x(x+7) = 0$$

$$x = 0 \qquad x + 7 = 0$$

$$x = 0 \qquad x = -7$$

Unit 3A – Solving Quadratic Equations by Factoring

Zero Factor Property:

If a and b are real numbers and if $ab = 0$,

then $a = 0$ or $b = 0$.

Examples:

$$(x-10)(3x-6)=0$$

$x-10=0$

$x-10+10=0+10$

$x=10$

$3x-6=0$

$3x-6+6=0+6$

$3x=6$

$\frac{3x}{3}=\frac{6}{3}$

$x=2$

Unit 3A – Solving Quadratic Equations by Factoring

Solving Quadratic Equations:

1) Write the equation in standard form.
2) Factor the equation completely.
3) Set each factor equal to 0.
4) Solve each equation.
5) Check the solutions (in original equation).

Unit 3A – Solving Quadratic Equations by Factoring

$$x^2 - 3x = 18$$

$$x^2 - 3x - 18 = 0$$

Factors of 18:

$1, 18 \quad 2, 9 \quad 3, 6$

$$(x+3)(x-6) = 0$$

$$x+3=0 \qquad x-6=0$$

$$x = -3 \qquad x = 6$$

$$(6)^2 - 3(6) = 18$$

$$36 - 18 = 18$$

$$18 = 18$$

$$(-3)^2 - 3(-3) = 18$$

$$9 + 9 = 18$$

$$18 = 18$$

Unit 3A – Solving Quadratic Equations by Factoring

If the Zero Factor Property is not used, then the solutions will be incorrect

$$x^2 - 3x = 18$$

$$x(x-3) = 18$$

$$x = 18 \qquad x - 3 = 18$$

$$x - 3 + 3 = 18 + 3$$

$$x = 21$$

$$(18)^2 - 3(18) = 18$$

$$324 - 54 = 18$$

$$270 \neq 18$$

$$(21)^2 - 3(21) = 18$$

$$441 - 63 = 18$$

$$378 \neq 18$$

Unit 3A– Solving Quadratic Equations by Factoring

$$x(x-4)=5$$

$$x^2-4x=5$$

$$x^2-4x-5=0$$

$$(x+1)(x-5)=0$$

$$x+1=0 \qquad x-5=0$$

$$x=-1 \qquad x=5$$

Unit 3A – Solving Quadratic Equations by Factoring

$$x(3x+7)=6$$

$$3x^2+7x=6$$

$$3x^2+7x-6=0$$

Factors of 3:

1, 3

Factors of 6:

1, 6 2, 3

$$(x+3)(3x-2)=0$$

$$x+3=0 \quad 3x-2=0$$

$$x=-3 \qquad 3x=2$$

$$x=\frac{2}{3}$$

Unit 3A – Solving Quadratic Equations by Factoring

$$9x^2 - 24x = -16$$

$$9x^2 - 24x + 16 = 0$$

$$(9 \textit{ and } 16 \textit{ are perfect squares})$$

$$(3x - 4)(3x - 4) = 0$$

$$3x - 4 = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

Unit 3A – Solving Quadratic Equations by Factoring

$$2x^3 - 18x = 0$$

$$2x\left(x^2 - 9\right) = 0$$

$$2x\left(x+3\right)\left(x-3\right) = 0$$

$$2x = 0 \qquad x+3 = 0 \qquad x-3 = 0$$

$$x = 0 \qquad x = -3 \qquad x = 3$$

Unit 3A – Solving Quadratic Equations by Factoring

$$(x+3)(3x^2-20x-7)=0$$

Factors of 3: $1, 3$ *Factors of* 7: $1, 7$

$$(x+3)(x-7)(3x+1)=0$$

$$x+3=0 \qquad x-7=0 \qquad 3x+1=0$$

$$x=-3 \qquad x=7 \qquad 3x=-1$$

$$x=-\frac{1}{3}$$

Unit 3A – Quadratic Equations and Problem Solving

A cliff diver is 64 feet above the surface of the water. The formula for calculating the height (h) of the diver after ***t*** seconds is: $h = -16t^2 + 64.$

How long does it take for the diver to hit the surface of the water?

$$0 = -16t^2 + 64$$

$$0 = -16\left(t^2 - 4\right)$$

$$0 = -16\left(t + 2\right)\left(t - 2\right)$$

$$t + 2 = 0 \qquad t - 2 = 0$$

~~$t = -2$~~ $\qquad t = 2$ seconds

The square of a number minus twice the number is 63. Find the number.

x is the number.

$$x^2 - 2x = 63$$

$$x^2 - 2x - 63 = 0$$

Factors of 63: $1, 63 \quad 3, 21 \quad 7, 9$

$$(x+7)(x-9) = 0$$

$$x+7=0 \qquad x-9=0$$

$$x=-7 \qquad x=9$$

Unit 3A – Quadratic Equations and Problem Solving

The length of a rectangular garden is 5 feet more than its width. The area of the garden is 176 square feet. What are the length and the width of the garden?

$l \cdot w = A$ The width is w. The length is $w+5$.

$(w+5)w = 176$

$w^2 + 5w = 176$

$w^2 + 5w - 176 = 0$

Factors of 176:

1, 176 2, 88 4, 44

8, 22 11, 16

$(w-11)(w+16) = 0$

$w - 11 = 0$ $w + 16 = 0$

$w = 11$ ~~$w = -16$~~

$w = 11$ feet $l = 11 + 5$

$l = 16$ feet

Unit 3A – Quadratic Equations and Problem Solving

Find two consecutive odd numbers whose product is 23 more than their sum?

Consecutive odd numbers: x $x+2.$

$$x(x+2)=(x+x+2)+23$$

$$x^2+2x=2x+25$$

$$x^2+2x-2x=2x+25-2x$$

$$x^2=25$$

$$x^2-25=25-25$$

$$x^2-25=0$$

$$(x+5)(x-5)=0$$

$x+5=0$

$x=-5$

$-5+2=-3$

$-5,-3$

$x-5=0$

$x=5$

$5+2=7$

$5,7$

Unit 3A – Quadratic Equations and Problem Solving

The length of one leg of a right triangle is 7 meters less than the length of the other leg. The length of the hypotenuse is 13 meters. What are the lengths of the legs? (*Pythagorean Th.*)

$$a^2 + b^2 = c^2$$

$$a = x \quad b = x - 7 \quad c = 13$$

$$x^2 + (x-7)^2 = 13^2$$

$$x^2 + x^2 - 14x + 49 = 169$$

$$2x^2 - 14x - 120 = 0$$

$$2(x^2 - 7x - 60) = 0$$

Factors of 60: $1, 60$ $2, 30$ $3, 20$ $4, 15$ $5, 12$ $6, 10$

$$2(x+5)(x-12) = 0$$

$$x + 5 = 0 \qquad x - 12 = 0$$

~~$x = -5$~~ $\qquad x = 12$

$a = 12$ meters

$b = 12 - 7 = 5$ meters